16. Considere as matrizes

$A = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$, $A^2 = A \,.\, A$, $A^3 = A^2.\,A$,........, $A^n = A^{n-1}.\,A$,........

Se d_n é o determinante da matriz A^n, então, a soma $d_1 + d_2 + d_3 + \ldots\ldots + d_n + \ldots\ldots$ é igual a

A) $\frac{1}{3}$.

B) $\frac{1}{2}$.

C) $\frac{2}{3}$.

D) $\frac{3}{4}$.

Assunto: Matrizes e P.G.

$$A = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \Rightarrow A = \frac{1}{2}.\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow A = \frac{1}{2}.I \Rightarrow A^n = \left(\frac{1}{2}\right)^n.I^n \Rightarrow$$

$$\Rightarrow A^n = \left(\frac{1}{2}\right)^n.I \Rightarrow A^n = \begin{bmatrix} \left(\frac{1}{2}\right)^n & 0 \\ 0 & \left(\frac{1}{2}\right)^n \end{bmatrix} \Rightarrow d_n = \left(\frac{1}{2}\right)^n.\left(\frac{1}{2}\right)^n - 0.0 \Rightarrow d_n = \left(\frac{1}{4}\right)^n$$

$$d_1 + d_2 + d_3 + \ldots + d_n + \ldots = \underbrace{\frac{1}{4} + \frac{1}{16} + \frac{1}{69} + \ldots + \left(\frac{1}{4}\right)^n + \ldots}_{\text{soma dos infinitos termos de uma P.G. com } a_1 = \frac{1}{4} \text{ e } q = \frac{1}{4}}$$

Aplicando a fórmula $S_\infty = \frac{a_1}{1-q}$, tem-se:

$$d_1 + d_1 + d_3 + \ldots + d_n + \ldots = \frac{\frac{1}{4}}{1 - \frac{1}{4}}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$= \frac{1}{3}$$

Item: A